ABSTRACTS

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INVITED TALKS

Stationary separability of quantum Hamiltonian systems

Maciej Błaszak

Adam Mickiewicz University in Poznań, Poland

We prove that many classical Hamiltonian systems that can not be separably quantized in the classical approach of Robertson and Eisenhardt can be separably quantized if we extend the class of admissible quantizations through a suitable choice of Riemann space adapted to the Poisson geometry of the system. Actually, we prove that for every quadratic in momenta Stäckel system for which separation relations are quadratic in momenta and are Laurent polynomials with respect to position coordinates, there exist infinitely many quantizations - parametrized by n arbitrary functions - that turn this system into a quantum separable Stäckel system.

Argument shift method and sectional operators: applications to differential geometry

Alexey Bolsinov

Loughborough University, United Kingdom

The talk is devoted to a surprising relationship between sectional operators, well known in the theory of integrable systems on Lie algebras, and curvature tensors of projectively equivalent Riemannian and Kaehler metrics of arbitrary signature. We demonstrate how using this relationship helps to solve a number of natural problems in differential geometry.

Painleve VI equations and the Poncelet polygons

Vladimir Dragovic

University of Texas, United States

A new method to construct algebro-geometric solutions of rank two Schlesinger systems is presented. For an elliptic curve represented as a ramified double covering of $\mathbb{CP}^1$, a meromorphic differential is constructed with the following property: the common projection of its two zeros on the base of the covering, regarded as a function of the only moving branch point of the covering, is a solution of a Painleve VI equation. This differential provides an invariant formulation of a classical Okamoto transformation for the Painleve VI equations. A generalization of this differential to hyperelliptic curves is also constructed. The research is motivated by a construction suggested by Hitchin in 1990's. The results are joint with V. Shramchenko.

Analytic solutions of the Somos 6 recurrence via hyperelliptic Prym varieties

Yuri N. Fedorov, A. N.W. Hone

Universitat Politcnica de Catalunya, Spain

Somos sequences are integer sequences generated by bilinear recurrence relations. They have appeared in number theory, statistical mechanics, as well as arising from reductions of bilinear PDE in the theory of discrete integrable systems. General bilinear recurrences of order $3, 4$, and 5 generate sequences of Fibonacci-type numbers, which can be written in terms of the sigma-function of appropriate elliptic curves.

This talk concerns the general form of the order 6 recurrence

$$\tau_{n+6}\tau_n = \alpha\tau_{n+5}\tau_{n+1} + \beta\tau_{n+4}\tau_{n+2} + \gamma\tau_{n+3}^2, \quad n \in \mathbb{N}$$

with arbitrary coefficients α, β, γ, which can be described as an integrable birational map φ on $\mathbb{C}^4$ having 2 independent algebraic integrals (the Somos 6 map). As was shown in [1], the solutions of φ are the first ones which are beyond genus one: they are parametrized by sigma-function of genus 2 curves.

Our goal is to reconstruct the sigma-function solutions of the Somos 6 map from the initial data: Namely, given the first 6 terms of the sequence $\{\tau_n\}$ we determine the equation of the corresponding hyperelliptic curve X and the translation vector $\mathbf{v} \in \mathrm{Jac}(X)$ in the sigma-function solution.

One of our main tools is a 3×3 Lax representation for the map φ, which was recently derived from the similar Lax pair for the discrete BKP equation, as was announced in [1]. The corresponding spectral curve S is trigonal of genus 4 having an involution σ with 2 fixed points. Then the 2-dimensional Jacobian of X, the complex invariant manifold of φ, is identified with a principally polarized Prym subvariety $\mathrm{Prym}(S, \sigma)$ of $\mathrm{Jac}(S)$.

To obtain an explicit algebraic description of $\mathrm{Prym}(S, \sigma)$ and, therefore, of X, we use the recent result of [2], which studies the general case of 2-fold coverings of hyperelliptic curves with 2 branch points.

References

[1] Hone, A. N. W. Analytic solutions and integrability for bilinear recurrences of order six. *Appl. Anal.* **89** (2010), no. 4, 473–492

[2] Levin, A. Siegel's theorem and the Shafarevich conjecture. *J. Théor. Nombres Bordeaux* 24 (2012), no. 3, 705–727

Z_N graded discrete Lax pairs and discrete integrable systems

Allan P. Fordy

University of Leeds, United Kingdom

We introduce a class of Z_N graded discrete Lax pairs, with $N \times N$ matrices, linear in the spectral parameter and classify the associated discrete integrable systems. Several well known examples belong to our scheme for $N = 2$ (discrete MKdV equation, Hirota's discrete sine-Gordon equation, discrete potential KdV, Schwarzian KdV equation), so many of our systems may be regarded as generalisations of these. Even at $N = 3$, several new integrable systems arise.

We also present continuous isospectral deformations these Lax pairs, giving compatible differential-difference systems, which play the role of continuous symmetries of the discrete systems. Master symmetries and their respective hierarchies can be constructed.

We present two non-local symmetries of our discrete systems, which give rise to the two-dimensional Toda lattice, with our nonlocal symmetries being the Bäcklund transformations and our discrete system being the nonlinear superposition formula (for the generic case).

This talk is joint work with Pavlos Xenitidis and based on the paper: `arXiv:1411.6059` [nlin.SI].

Cluster structure on the Drinfeld double of $\mathrm{GL}(n)$

Michael Gekhtman

University of Notre Dame, United States

We describe a generalized cluster structure on the Drinfeld double of the group $\mathrm{GL}(n)$ equipped with the standard Poisson Lie bracket. This generalized cluster structure restricts to the known cluster structure on $\mathrm{GL}(n)$ and to a generalized cluster structure on its dual Poisson-Lie group. This work is a part of the joint project with M. Shapiro and A. Vainshtein on constructing cluster structures compatible with Belavin-Drinfled Poisson-Lie brackets on simple Lie groups.

Invariant classification and limits of superintegrable systems

Jonathan Kress

University of New South Wales, Australia

A classical Hamiltonian system possessing more globally defined conserved quantities than degrees of freedom is said to be superintegrable. Such systems possessing the maximum number of conserved quantities quadratic in the momenta have been much studied because of their connection with separation of variables and special functions. In two dimensions it has been shown that singular limits between "non-degenerate" superintegrable systems mirror limits between orthogonal polynomials in the Askey scheme. As a first step in extending this to higher dimensions, all second order superintegrable systems with a "non-degenerate" potential have recently been classified and shown to arise by singular limits from a generic system on the 3-sphere. This classification and the relationships between the systems, which is joint work with Joshua Capel and Sarah Post, will be discussed.

On the integrability of some rank 2 sub-Riemannian structures

Boris Kruglikov

University of Tromsø, Norway

We will discuss integrability of the Hamiltonian systems corresponding to some lower-dimensional Carnot groups. The work is joint with Andreas Vollmer.

Bifurcations in 3 d.o.f. integrable Hamiltonian systems

Lev Lerman

Lobachevsky State University of Nizhny Novgorod, Russia

When dealing with integrable Hamiltonian systems on some symplectic manifold we encounter the problem of studying bifurcations. This is due to the fact that all singular orbits of the related Poisson action (of dimension lesser than the half of the manifold dimension) are met in families in such systems. For instance, periodic orbits being 1-dimensional orbits of the induced Poisson action belong to a 1-parameter families, 2-dimensional Lagrangian tori belong to 2-parameter families, etc. This implies that moving along the family we can face an orbit being more degenerate (in transverse direction) than neighboring orbits and thus we have to expect some branching the family. Also bifurcations are met when studying families of integrable systems, then their parameters play a similar role. It is important to stress that common tool to study integrable systems uses some assumptions on the linearized system at the related Poisson orbits (like to be a Cartan algebra for the related set of commuting integrals, etc). Such properties are usually violated at the bifurcation and one needs to use another tool to study the related orbit structure.

I intend to discuss this topic for 3 degree of freedom integrable Hamiltonian systems. In this case (if no outer parameter exist) the related integrable system can contain 1-parameter families of periodic orbits and 2-parameter families of Lagrangian 2-tori. Thus one can meet degenerations of codimension 1 and two. Related structures of Liouville foliations in saturated neighborhoods of the degenerated orbits and their "perestroika's" will be presented.

Traveling waves in chains of oscillators

Mark Levi

Penn State University, United States

I will discuss some results, old and new, on travelling waves in chains of coupled oscillators (such as pendula), and will pose some open questions.

Integrable geodesic flows on 2-torus and the systems of hydrodynamical type

Andrey Mironov

Nowosybirsk State University & Moscow State University, Russia

We study quasi-linear system of partial differential equations which describes the existence of the polynomial in momenta first integral of the integrable geodesic flow on 2-torus. We prove that in the case of integrals of degree three and four the system is equivalent to a single equation of order 3 and 4 respectively. Remarkably the equation for the case of degree $n = 4$ has variational meaning: it is Euler-Lagrange equation of a variational principle. This equation for $n = 4$ has formal double periodic solutions as a series in a small parameter.

Integrable geodesic flows on diffeomorphism groups

Gerard Misiolek

University of Notre Dame, United States

I will describe examples of integrable nonlinear PDE on groups of diffeomorphisms which arise in hydrodynamics and information geometry and fit within the framework of V. Arnold.

Dynamics of rolling and sliding rigid bodies: Jelletts egg, tippe top (TT), disc (sRD)

Stefan Rauch-Wojciechowski

Dept. of Mathematics, Linköping University, Sweden

Equations of motion for purely rolling axially symmetric rigid bodies have 4 degrees of freedom and are completely integrable (Routh, Chaplygin). When sliding is allowed there are 2 additional degrees of freedom for motion of centre of mass, equations are dissipative and integrability is lost. Analysis of this nonlinear dynamical systems is difficult and progress is limited. The main tools in study of global dynamics is the monotonously decreasing energy function, theorems on stability of asymptotic solutions and the use of LaSalle' type theorems. An additional useful property is that the underlying purely rolling problem is integrable and this can be used for studying dynamics with sliding as in the case of the Tippe top where it helped to explained oscillatory behavior of inverting solutions of TT. The asymptotic solutions provide also a useful framework for numerical sampling of solutions to get an idea of what happens at different initial condition regimes. I will present numerical simulations of inverting solutions of the tippe top. They display new interesting features; initiation of inversion requires reaching a threshold value of the angular velocity and synchronization of remaining variables. Remarkably it is possible to discern a (dynamically distinguished) starting and ending time for the inversion. In simulations of a rolling and sliding disc, it appears that solutions above a certain energy threshold go asymptotically to one of the tumbling solutions (with the disc is rolling around a fixed centre of mass). In certain regions of initial conditions the outcome is extremely sensitive to small changes of initial data.

Pentagram map

Michael Shapiro

Michigan State University, United States

Cluster algebra is formed by a collection of sets of rational functions (clusters) that forms a graph. Clusters connected by an edge satisfy simple relations. A path in the graph of clusters determines a discrete dynamics. We will discuss examples of such cluster dynamics which lead to integrable discrete dynamical systems (the main example, is the pentagram map on the space of twisted n-gons in the projective plane, which sends an n-gon to the n-gon formed by its short diagonals)

Linear symplectic maps and billiard dynamics

Dmitry V. Treschev

Steklov Mathematical Institute of Russian Academy of Science & Moscow State University, Russia

We study billiard systems (including multi-dimensional) whose dynamics is locally conjugated to linear

Cubically superintegrable systems in dimension 2 and their geodesic flow

Galliano Valent

LPMP: Laboratoire de Physique Mathématique de Provence, France

(Joint work with C. Duval and S. Shevchishin)

We have obtained, in local coordinates, the explicit form of the two-dimensional superintegrable systems of Matveev and Shevchishin involving a linear and two cubic integrals. This allows to study their global structure, showing that some systems are globally defined either on the manifold S^2 or on Tannery's orbifold. While on the manifold the geodesic flow is Zoll (all the geodesics are closed and of the same length), on the orbifold the geodesic flow is either Zoll or Tannery (all geodesics are still closed but not necessarily of the same length).

Integrability via reversibilty

Maciej P. Wojtkowski

Faculty of Mathematics and Computer Science, University of Warmia and Mazury in Olsztyn, Poland

A class of left-invariant second order reversible systems with functional parameter is introduced which exhibits the phenomenon of robust integrability: an open and dense subset of the phase space is filled with invariant tori carrying quasi-periodic motions, and this behavior persists under perturbations within the class. Real-analytic volume preserving systems are found in this class which have positive Lyapunov exponents on an open subset, and the complement filled with invariant tori.

This class of dynamical systems is developed from the examples of integrable geodesic flows with positive topological entropy discovered by Butler, Bolsinov and Taimanov.

A trip searching for integrability criteria over 120 years

Haruo Yoshida

National Astronomical Observatory of Japan, Japan

One of the fundamental questions in the field of integrable systems is to find an algorithm, which tells us whether a given system is integrable or not. In the last 30 years there has been a considerable progress in this direction, which has a root in the work of S. Kowalevskaya about more than 120 years ago. Her work suggested that there is a hidden relation between branching of solutions as functions of the complex time and integrability of the system.

In this talk I shall review these progress as the order of time. Among others, special emphasis will be made on the Hamiltonian system with a 2D homogeneous polynomial potential of the form

$$H = \frac{1}{2}\left(p_1^2 + p_2^2\right) + V(q_1, q_2),$$

for which algorithmic necessary conditions for integrability have been obtained by S.L. Ziglin, J. J. Morales-Ruiz, J. P. Ramis, and A. J. Maciejewski etc.

On the notion of integrability for stochastic dynamical systems

Nguyen Tien Zung

Universit Paul Sabatier Toulouse III, France

We extend the notion of integrability to stochastic dynamical systems, and discuss its relation with symmetry, reduction, quantum and classical integrability, integrable metrics, etc.

The Hess–Appelrot system

Henryk Żołądek

University of Warsaw Poland

We study the Hess–Appelrot case of the Euler-Poisson system which describes dynamics of a rigid body about a fixed point. We prove existence of an invariant torus S which supports hyperbolic or parabolic or elliptic periodic or elliptic quasi-periodic dynamics. In the elliptic cases we study the question of normal hyperbolicity of the invariant torus in the case when the torus is close to a “critical circle”. It turns out that the invariant torus is normally hyperbolic when the torus is close to a “critical circle” and the motion is $1 : q$ resonant. Next, we consider perturbation of the Hess–Appelrot system (within the Euler–Poisson class) near the above situation. We prove existence of an invariant surface close to S and we study limit cycles on the perturbed surface. We estimate the number of such cycles by analysis of some non-standard Melnikov integrals.

REGULAR TALKS

Euler equations on the general linear group, cubic curves, and inscribed hexagons

Konstantin Aleshkin

Lomonosov MSU, Russia

We study integrable Euler equations on the Lie algebra gl(3, R) by interpreting them as evolutions on the space of hexagons inscribed in a real cubic curve.

Effective bounds in E. Hopf rigidity for billiards and geodesic flows

Michael Bialy

Tel-Aviv University, Israel

We show that in some cases the E. Hopf rigidity phenomenon allows quantitative interpretation. More precisely, we estimate from above the measure of the set $\mathcal{M}$ swept by minimal orbits. These estimates are sharp, i.e. if $\mathcal{M}$ occupies the whole phase space we recover the E. Hopf rigidity. We give these estimates in two cases: the first is the case of convex billiards in the plane, sphere or hyperbolic plane. The second is the case of conformally flat Riemannian metrics on a torus.

Modified Laplace-Beltrami quantization of extended systems

Claudia Maria Chanu

University of Turin, Italy

A quantization procedure of certain natural Hamiltonian systems with quadratic first integrals is developed. Necessary and sufficient conditions are determined for the modified quantization of classical natural Hamiltonians with a quadratic first integral recursively generated by a differential operator, together with compatibility conditions allowing a simultaneous modified quantization for additional first integrals. Both the cases of conformally flat manifolds and non-conformally flat manifolds are examined.

Integrable polynomial homogeneous potentials in the plane

Thierry Combot

University of Burgundy, France

We present an almost complete classification of integrable polynomial homogeneous potentials in the plane. The classification work for degree $3, 4$ was already done by A. J. Maciejewski and M. Przybylska, and a computer method made it possible to extend their approach for an arbitrary degree. This allowed to deal with homogeneity degree ≤ 5. For $k \geq 5$, higher variational techniques are used and allow to deal with all other degrees. The few cases for which integrability status is still unknown is a family potentials $V_{n,k}$ without any Darboux points and a few examples which are for the moment too complicated to analyze higher variational equations. The family $V_{n,k}$ presents very special symmetries which allow some integrability analysis to be done through the Morales–Ramis method, but still seem to give no integrability conditions.

Nonintegrability of the optimal control problem for n-level quantum systems

Guillaume Duval and Witold Respondek

Laboratoire de Mathématiques, LMI, INSA de Rouen, France

We study the problem of optimal laser-induced population transfer in n-level quantum systems. This problem can be represented as a sub-Riemannian problem on $\mathrm{SO}(n)$, and it is known (Boscain, Charlot, Gauthier) that for $n = 2$ and $n = 3$ the Hamiltonian system associated with Pontryagin maximum principle (PMP) is integrable. We will show that this changes completely for $n \geq 4$. Namely, the adjoint equation of the PMP does not possess any meromorphic first integral independent of the Hamiltonian on the levels of the Casimir function. To this aim, we will use the differential Galois framework to the integrability.

From compact semitoric systems to Hamiltonian S^1-actions and back

Sonja Hohloch

Universiteit Antwerpen, Belgium

Every semi-toric system induces a Hamiltonian S^1-action on the manifold by “forgetting” the R-valued flow parameter. Effective Hamiltonian S^1-actions on compact 4-manifolds have been classified by Karshon by means of so-called “labeled directed graphs”.

In a joint work with S. Sabatini and D. Sepe, we linked Pelayo $\&$ Vu Ngoc’s classification of semi-toric systems to Karshon’s classification of Hamiltonian S^1-actions. More precisely, we show that only 2 of the 5 invariants are necessary to deduce the Karshon graph of the underlying S^1-action.

Together with S. Sabatini, D. Sepe and M. Symington, we showed how to “lift” an effective Hamiltonian S^1-action on a compact 4-manifold to a semi-toric system.

In this talk, we give an introduction to semi-toric systems and Hamiltonian S^1-actions and sketch parts of our constructions.

Singular fibers of integrable systems: a combinatorial point of view

Anton Izosimov

University of Toronto, Canada

It is well-known that regular fibers of most integrable systems can be described as open subsets in Abelian varieties. The general question I am going to address is how to generalize this description to singular fibers.

I will concentrate on the integrable system defined on the dual of the Lie algebra $\mathrm{gl}(n)$ by shifting the invariants of the coadjoint representation. The fibers of this system are enumerated by plane algebraic curves, and each regular fiber is (modulo conjugation by diagonal matrices) an open subset in the Jacobian of the corresponding curve. I will explain what happens when one considers a fiber corresponding to a singular curve. Such fibers turn out to have interesting combinatorics related to convex polytopes, orientations of graphs etc.

The talk will be mostly combinatorial. No background in algebraic geometry is assumed.

The Clebsch top

Franco Magri

Università degli Studi di Milano-Bicocca

I will show a new way of solving the equations of motion of the Clebsch Top by separation of variables, one hundred years or more after Kotter.

An algebraic geometric classification of superintegrable systems

Konrad Schöbel

Friedrich-Schiller-Universität Jena, Germany

Superintegrable systems are integrable dynamical systems possessing a maximum number of constants of motion. We argue that the classification of superintegrable systems is essentially an algebraic geometric problem. Motivated by recent results relating the classification of separable systems to moduli spaces of algebraic curves, we show that the space of (non-degenerated 2D maximally) superintegrable systems is a projective variety and describe its geometry. In particular, we relate the classification of superintegrable systems to moduli spaces of hyperplane configurations, which provides a canonical classification scheme. Several generalisations and applications will be proposed. This is a joint work with Jonathan Kress.

New integrable $(3+1)$-dimensional systems from compatibility of contact Hamilton–Jacobi equations

Artur Sergyeyev

Silesian University in Opava, Czech Republic

We present a generalization of a well-known construction yielding many $(2+1)$-dimensional integrable dispersionless systems as compatibility conditions for pairs of compatible nonstationary Hamilton–Jacobi equations with one degree of freedom.

Namely, we construct a broad new class of $(3+1)$-dimensional integrable dispersionless systems that arise as compatibility conditions of nonstationary contact Hamilton–Jacobi equations.

Please see `arXiv:1401.2122` for details.

Differential invariants for cubic integrals of geodesic flows on surfaces

Vsevolod Shevchishin

We construct differential invariants that vanish if and only if the geodesic flow of a two-dimensional metric admits an integral of third degree in momenta with a given Birkhoff-Kolokoltsov 3-codifferential.

Falling motion of a circular cylinder interacting dynamically with point vortices

Sergei Sokolov

Institute of Machines Science named after A.A.Blagonravov of the Russian Academy of Sciences, Russia

The problem of falling motion of a body in fluid has a long history and was considered in a series of the classical and modern papers. Some of the effects described in the papers, such as periodic rotation (tumbling), can be encountered only in viscous fluids and thus demand for their proper treatment the use of the Navier–Stokes equations with boundary conditions specified on the bodys surface. As a rule, such problems are hardly amenable to analytical analysis and can be addressed only numerically. Another approach is to use (instead of the exact Navier–Stokes equations) some phenomenological ODE models which capture the viscous effects qualitatively. In this paper we study the influence of the vorticity on the falling body in a trivial setting: a body (circular cylinder) subject to gravity is interacting dynamically with N point vortices. The circulation around the cylinder is not necessarily zero. So the model we consider here is exact and, at the same time, not so despairingly complex as most of the existing models are. The dynamical behavior of a heavy circular cylinder and N point vortices in an unbounded volume of ideal liquid is considered. The liquid is assumed to be irrotational and at rest at infinity. The circulation about the cylinder is different from zero. The governing equations are presented in Hamiltonian form. Integrals of motion are found. Allowable types of trajectories are discussed in the case of single vortex. The stability of finding equilibrium solutions is investigated and some remarkable types of partial solutions of the system are presented. Poincare sections of the system demonstrate chaotic behavior of dynamics, which indicates a non-integrability of the system. In a case of zero circulation using autonomous integral we can also reduce the order of the system by one degree of freedom. Unlike nonzero circulation and the absence of vortices when the cylinder moves inside a certain horizontal stripe it is shown that in a presence of vortices and with circulation equal to zero vertical coordinate of the cylinder is unbounded decreasing. We then focus on the numerical study of dynamics of our system. In a case of zero circulation trajectories are noncompact. The different kinds of the scattering function of the vortex by cylinder were obtained. The form of these functions argues to chaotic behavior of the scattering which means that an additional analytical integral is absent.

Stability analysis for certain generalized free rigid bodies

Daisuke Tarama

Department of Mathematics, Kyoto University, Japan

The integrability and the stability of the equilibria for the ordinary free rigid body dynamics on $SO(3)$ are well known. This completely integrable system was generalized to arbitrary complex semi-simple Lie algebras by Mishchenko and Fomenko around 1980, as well as its normal and compact real forms. Recently, certain researches have been done on the stability of the equilibria for these generalized free rigid body dynamics. In particular, Bolsinov, Oshemkov, and Izosimov have given a systematic approach to deal with the integrability and the non-degeneracy of the isolated equilibria on a generic symplectic leaf for bi-Hamiltonian systems, by means of which the stability analysis has been done in the $SO(n)$ case by Izosimov. In this talk, their methods, combined with the structure theory of semi-simple Lie algebras, are applied to the stability analysis for the Mishchenko-Fomenko free rigid bodies on the normal and the compact real forms of complex semi-simple Lie algebras, $U(n)$ free rigid bodies, and Bloch-Iserles systems. It is shown that the isolated equilibria on a generic adjoint orbit are either uniformly elliptic or uniformly hyperbolic for these systems.

Killing tensors with non-vanishing Haantjes torsion and integrable systems

Andrey Tsiganov

Saint Petersburg State University, Russia

Second order integrable Killing tensor with simple eigenvalues and vanishing Haantjes torsion is the key ingredient in construction of integrable in the Liouville sense systems of the Stackel type. We present a few examples of the integrable systems on three-dimensional Euclidean space associated with the second order Killing tensors possessing non-trivial torsion. Integrals of motion for these integrable systems are the second and fourth order polynomials in momenta, which are constructed using special family of the Killing tensors.

Determining algorithmically the number of polynomial integrals for a Hamiltonian flow

Andreas Vollmer

University of Jena, Germany

The talk will present two results and the methods of how to obtain them. One is integrability for certain stationary and axially symmetric Einstein metrics. Another is the integrability in sub-Riemannian Carnot groups.

Both results are based on a computer implementation of the Cartan-Khler prolongation procedure. An algorithm will be given for analyzing integrability effectively for specifically given Hamiltonian flows. It allows to find additional polynomial integrals, or to prove their nonexistence. It will be demonstrated how simple modifications of the algorithm can substantially improve its efficiency.

One of the examples to be presented is related to joint research with B. Kruglikov and G. Lukes-Gerakopoulos where the algorithm has been applied to prove nonexistence of an additional integral in certain sub-Riemannian Carnot groups. The talk will focus on the method in its own right, while other aspects of this research will be presented by B. Kruglikov during his talk. The algorithm can be applied very generally and is in no way restricted to one specific example. A further example will be presented: Stationary axially symmetric Ricci-flat Einstein metrics. For a subclass of these metrics, an improved algorithm will be described and illustrated for a metric which allows to prove with the algorithm nonexistence of an additional polynomial integral up to degree 11.

POSTERS

Integrability of the Szekeres system

Anna Gierzkiewicz

Agriculture University in Kraków and Jagiellonian University, Poland

A Silent Universe is a dust spacetime with unit velocity vector field $\vec{u}$ with no rotation ($\omega_{ab} = 0$) and Weyl tensor purely electric with respect to $\vec{u}$, i.e. its magnetic part vanishes ($H_{ab}(\vec{u}) = 0$). This notion has been introduced by Matarrese *et al.*during the study of evolution of the early universe. Using hydrodynamical formalism, one derives six evolution equations with six variables: the expansion rate θ, the shear rate σ_{ab}, the energy density ρ and the electric part of Weyl tensor E_{ab}. The Szekeres model reduces the system to four equations by setting $E = \frac{1}{2}(E_1 + E_2)$ and $\sigma = \frac{1}{2}(\sigma_1 + \sigma_2)$:

$$\begin{cases} \rho' = -\Theta\rho \\ \Theta' = -\frac{1}{3}\Theta^2 - 6\sigma^2 - \frac{1}{2}\rho \\ \sigma' = \sigma^2 - \frac{2}{3}\Theta\sigma - E \\ E' = -3E\sigma - \Theta E - \frac{1}{2}\rho\sigma \end{cases} .$$

From purely mathematical point of view, the Szekeres system is a four-dimensional system of first-order, ordinary differential equations, with non-linear, but polynomial (quadratic) right hand side. It is an exact solution of the Einstein equations, which is inhomogeneous and has no symmetries. It has been used, among others, in modelling the early Universe or the evolution of galaxy super-clusters. I have studied the integrability of this system in the sense of finding its first integrals. The Darboux polynomials method allowed me to find two independent rational first integrals. One can obtain the last independent integral via Jacobi's last multiplier method. To find such a multiplier for Szekeres system, I have modified Goriely's method for quasi-monomial systems. Therefore, the Szekeres system is completely integrable.

The trigonometric BC(n) Sutherland system: action-angle duality and applications

Tamás F. Görbe

University of Szeged, Hungary

Action-angle duality for the trigonometric BC(n) Sutherland system is explored via Hamiltonian reduction [1]. Consequently, various features such as equilibrium, degeneracy, and connection to a family of commuting Hamiltonians found by van Diejen are elucidated [2].

References

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[2] T. F. Görbe, L. Feher, Equivalence of two sets of deformed Calogero-Moser Hamiltonians, arXiv:1503.01303 [math-ph]; submitted, 2015.

Straightenings of Lie supergroups

Óscar Guajardo

IMUNAM, Mexico

After a quick overview of a differential-geometric approach to supermanifolds, the main result of which is a flowbox theorem for these spaces, I'll focus on applying this approach to the theory of Lie supergroups. In particular we'll talk about "straightenings", understood as isomorphisms of an arbitrary supermanifolds with a split one.

Topology of integrable Hamiltonian systems on revolution surfaces

Elena Kantonistova

Moscow State University, Russia

In this work we study topology of Integrable Hamiltonian Systems on rebolution surfaces under the action of smooth potential field. We construct bifurcation diagram and bifurcation complex and then, using it, we calculate marked molecules (or Fomenko–Zieschang invariants) that fully determine the Liouville fibration of the phase space of the system.

As a result, we prove the topological classification theorem for studied systems.

Then we explain how to construct any system with potential using a system without potential (well-known geodesic flow on revolution surface).

On projective differential geometry of three-regular family of hyperplane elements

Artur Kuleshov

Immanuel Kant Baltic Federal University, Russia

In multidimensional projective space a family B of hyperplane elements is considered. The problem of constructing an intrinsic clothing of such a family is posed. The solution of this problem is obtained in the general case characterized by three-regularity condition. This solution is based on the procedure of reduction of the frame bundle attached to the family B. Two applications of the result are pointed. Firstly, in the special case the canonical almost product structure on the family is found. Secondly, in the general case two linear connections are attached intrinsically to the family. The expressions of the curvature tensors of these connections are found.

The moduli space of semitoric systems

Joseph Palmer

University of California, San Diego, United States

A semitoric integrable system is a 4-dimensional integrable system in which one of the integrals generates a Hamiltonian S^1-action. Pelayo-Vu Ngoc have recently proved a classification theorem for semitoric systems analogous to Delzant's classification of toric systems. In this poster I present a method of defining a natural metric space structure, and thus topology, on the moduli space of semitoric systems by exploiting this classification. This space is not connected but we describe its connected components. To show that these components are themselves connected we introduce semitoric fans (in analogy to toric fans) and a new method related to $\mathrm{SL}(2, Z)$ to understand toric and semitoric fans. The portion on semitoric fans and connectedness is joint with D.M. Kane and A. Pelayo.

Topology of dynamics of a nonhomogeneous rotationally symmetric ellipsoid on a smooth plane

Georgii Sechkin

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In our case, there is one holonomic constraint: the height of the center of mass above the plane is determined by the orientation of the central principal axes. Thus, the number of degrees of freedom is reduced to five. Let us write the equation in Euler's form using $f' = [f; H]$, where H is the Hamiltonian of $e(3)$. Then in the (S, R) coordinates: $S' = [S; H], R' = [R; H]$, we get the following first integrals: $H = \frac{1}{2}\sum_i (S_i^3/A_3) + U$ and $K = S_3$.

Fomenko and Zieshang proved that the two systems are equivalent in Liouville if they match same invariants. In my present work, I have constructed invariants for my system.

Zhukovsky found a generalization of Eulers integrable case, with Hamiltonian $H = S_1^2 + S_2^2 + S_3^2$. Additional integral is the same as in the Euler‘s case: $K = 1/2\sum_i (S_i^3 + l_i/J_3)$.

Theorem. This system of a symmetric ellipsoid on a smooth plane can be fully embedded, in the sense of Liouville equivalence, in the Zhukovsky system of a heavy gyrostat. So, for all values of the ellipsoid problem parameters there exist such parameters of Zhukovsky system for which invariants coincide.

Periodic orbits in the Kepler-Heisenberg problem

Corey Shanbrom

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The Kepler problem is among the oldest and most fundamental problems in mechanics. It has been studied in curved spaces, such as the sphere and hyperbolic plane. Here, we formulate the problem on the Heisenberg group, the simplest sub-Riemannian manifold. We take the sub-Riemannian Hamiltonian as our kinetic energy, and our potential is the fundamental solution to the Heisenberg sub-Laplacian. The resulting dynamical system is known to contain a fundamental integrable subsystem. We discuss the use of variational methods in proving the existence of periodic orbits with k-fold rotational symmetry for any odd integer k greater than one, and show approximations for $k = 3, 5$. Numerical methods which take advantage of the variational formulation are used to find approximate solutions having the sought-after symmetries. The sub-Riemannian structure on the Heisenberg group allows us to parametrize the optimization problem in terms of a single complex-valued curve, the Fourier decomposition of which lends itself to a particularly simple expression of the symmetry conditions.

On the monodromy of almost toric fibrations on the complex projective plane

Gleb Smirnov

International School for Advanced Studies, Italy

We describe the monodromy for almost toric Lagrangian fibrations (in the sense of Symington) on the complex projective plane. This problem is reduced to solving the celebrated Markov equation.
`http://arxiv.org/abs/1503.04458`

Note on integrability of certain homogeneous Hamiltonian systems

Wojciech Szumiński

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We investigate a class of natural Hamiltonian systems with two degrees of freedom. The kinetic energy depends on coordinates but the system is homogeneous. Thanks to this property it admits, in general case, a particular solution. Using this solution we derive necessary conditions for the integrability of such systems investigating differential Galois group of variational equations.

Non-integrability of constrained double pendula

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We consider two special types of double pendula, with the masses restricted to various surfaces. In order to get quick insight into the dynamics of considered systems the Poincaré cross sections as well as bifurcation diagrams have been used. The numerical computations show that both models are chaotic which suggest that they are not integrable. We give an analytic proof of this fact checking the properties of the differential Galois group of the systems variational equations along a particular non-equilibrium solution.

Singularities of 4-dimentional integrable Hamiltonian systems: the saddle-saddle case

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For any singular point of integrable Hamilton's system of two degrees of freedom there is an invariant of Liouville equivalence named circular molecule. If the type of singular point is center-saddle, center-center, or focus-focus, then the marked circular molecule is complete invariant of Liouville equivalence, but in the saddle-saddle case it is not true. To demonstrate this, Grabezhnoy constructed two examples of different 4-dimensional singularities with coincided marked circular molecule. These two examples there are the only ones known nowadays. In our talk we present an infinite series of different 4-dimensional singularities with coincided circular molecule. Let us also mention that in the saddle-saddle case any singularity is Liouville equivalent to a direct product of atoms factorized by a discrete group action. In our talk we describe the relation between the molecule marks and that group.

About the image of the moment map of semi-toric systems

Christophe Wacheux

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Almost-toric systems of complexity c are integrable Hamiltonian systems on a $2n$-dimensional compact symplectic manifold for which the moment map F yields a global $(n-c)$-torus action. Moreover, we ask that all critical points are non-degenerate in a certain sense, and without hyperbolic component in its quadratic part. If $c=0$, it is called toric, if $c=1$, it is called semi-toric. In 1982, Atiyah, and Guillemin $\&$ Sternberg, proved independently that for the moment map of the torus action, the fibers are connected and its image is a rational convex polytope. Delzant gave a constructive proof that this polytope characterizes entirely the system, thus providing a complete classification of toric systems.

This classification do not hold anymore for $c>0$. Yet for semi-toric systems in dimension $2n=4$, Vu Ngoc and Pelayo managed to recover a classification "á la Delzant". In our talk we will present a semi-toric version of Atiyah–Guillemin $\&$ Sternberg theorem. If time permits, we shall describe how we shall extend the classification of semi-toric systems in higher dimension.

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